Daley and Green: Waiting for News in the Market for Lemons

Econometrica (2012)

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LSE Economic Theory Reading Group

Waiting for News in the Market for Lemons: Two Periods

Akerlof's Market for Lemons: The Set-up

Markets with information asymmetries:
used cars (lemons vs. peaches), corporate bonds, mortgages, oil wells

Set-up: A seller has a car of type θ—good (H) or bad (L).
Buyers guess: Probability π_0 it's good (H).

Assumptions

- Seller's value: $K_L < K_H$
- Buyers' value: $V_L < V_H$
- Gains-from-trade: $V_H > K_H$, $V_L > K_L$.

Adverse Selection: Those most eager to trade ($K_L < K_H$) are the least attractive to trade with ($V_L < V_H$)

Problem: Buyers can't tell H from L.

Competitive Equilibrium

In equilibrium, the price is equal to the expected value of those seller's willing to trade.

Definition

A competitive equilibrium is a price P where:

- Sellers sell if $P \geq K_\theta$ (price exceeds their value).
- Buyers pay their expected value:

$$P = \mathbb{E}^{\pi_0}\left[V_\theta \middle| P \geq K_\theta\right]$$

- **Two outcomes**:
  1. $P = V_L$: Only lemons sell.
  2. $P = \pi_0 V_H + (1 - \pi_0) V_L$: Both sell (average price).

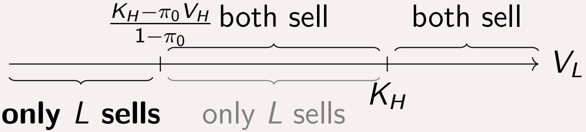
Equilibrium: Who Trades?

Case 1: Big Gains ($V_L \geq K_H$): Unique equilibrium

- both types sell at $P = \mathbb{E}[V_\theta]$.

Case 2: Small Gains ($V_L < K_H$): Two equilibrium candidates

- $P = V_L$ always an equilibrium—H types stay off the market.
- both selling is an eq if $P = \pi_0 V_H + (1 - \pi_0) V_{=} \mathbb{E}[V_\theta] \geq K_H$.



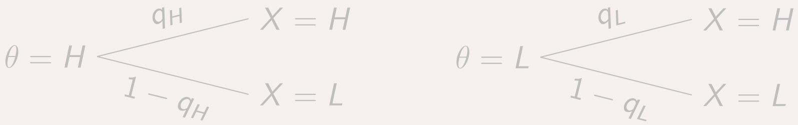
Market Failure: If $\mathbb{E}[V_\theta] < K_H$, high types don't trade.

Daley and Green, 2012: Waiting for News

What if sellers can sell at a later date and the market learns?

Set-up (toy model)

- Two periods: Trade now (period 0) or later (period 1).
- Absent period 0 trade, a signal X reveals good H or bad L news. q_θ is the probability that the signal reveals good news given θ:



- Assume that $q_H > \frac{1}{2}$ and $q_H > q_L$.
- No discounting (in toy model): Payoff is $P - K_\theta$ anytime.

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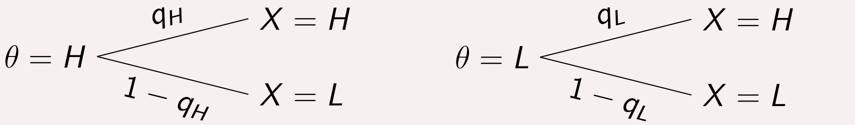

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Strategies and Beliefs: Who Sells When?

Strategy: S_θ = probability type θ sells in period 0.

No Period 0 trade? Market learns in two ways:

- If only $\theta = L$ sells, observing no trade means $\theta = H$.
- Buyers learn from signal X.

Beliefs update: belief after signal $X \in \{L, H\}$ and no trade is $\pi_X = Pr[\theta = H | \text{no trade}, X]$:

$$\pi_L = \frac{\pi_0(1-q_H)(1-S_H)}{\pi_0(1-q_H)(1-S_H) + (1-\pi_0)(1-q_L)(1-S_L)}$$

$$\pi_H = \frac{\pi_0 q_H(1-S_H)}{\pi_0 q_H(1-S_H) + (1-\pi_0)q_L(1-S_L)}$$

Seller Continuation Value: Wait or Sell?

Sellers anticipate a random **price path**:

- Period 0: P_0.
- Period 1: $X = H$ (prob. q_θ) $\rightarrow P_H$, $X = L$ (prob. $1 - q_\theta$) $\rightarrow P_L$.

Value of waiting:

$$F_\theta = q_\theta[P_H - K_\theta]_+ + (1 - q_\theta)[P_L - K_\theta]_+$$

where $[x]_+ = \max\{x, 0\}$: Profit if worth selling.

Optimal Stopping: Sell in period 0 if $P_0 - K_\theta \geq F_\theta$.

Equilibrium Definition

Definition

A competitive equilibrium is a price path (P_0, P_L, P_H) satisfying

- **Seller Optimality**: In period 1, sell if $K_\theta \leq P_X$. In period 0:

$$S_\theta \begin{cases} \in (0,1) & \text{if } F_\theta = P_0 - K_\theta \\ = 0 & \text{if } F_\theta > P_0 - K_\theta \\ = 1 & \text{if } F_\theta < P_0 - K_\theta \end{cases}$$

- **Consistent Beliefs and Zero Profit**: In period 1: $P_X = \mathbb{E}^{\pi_X}\left[V_\theta \middle| P_X \geq K_\theta\right]$. In period 0, unless $S_L = S_H = 0$,

$$P_0 = \mathbb{E}[V_\theta|\text{trade in } 0] = \frac{\pi_0 S_H V_H + (1-\pi_0) S_L V_L}{\pi_0 S_H + (1-\pi_0) S_L}$$

- **No Unrealized Deals**: If $S_L = S_H = 0$, then $F_L \geq V_L - K_L$.

Pareto-Dominated Equilibrium

In static Akerlof there exists a Pareto-dominated equilibrium.

$P = V_L$ is an equilibrium, even when $\mathbb{E}[V_\theta] > K_H$.

With learning, possibly $P_H < P_L$ even though $\pi_H \geq \pi_L$.

(only $\theta = L$ trades under $X = H$, but both types trade under $X = L$).

Rule out via **monotone price** (no Pareto-domination) **refinement**:

Definition (monotone price refinement)

Equilibrium period 1 prices satisfy $P_H \geq P_L$, strictly so if $S_L \cdot S_H < 1$.

Lemma

In any price-monotone equilibrium, $S_L < 1 \Rightarrow S_H = 0$.

Proof.

Type L weakly prefers not to trade in period 0 if $0 \geq P_0 - K_L - F_L$:

$$0 \geq P_0 - K_L - F_L = P_0 - K_L - q_L[P_H - K_L]_+ - (1 - q_L)[P_L - K_L]_+$$

Then note that, since $P_H > P_L$ and $q_H > q_L$,

$$\begin{aligned} P_0 - K_L - F_L = & P_0 - K_L - q_L[P_H - K_L]_+ - (1 - q_L)[P_L - K_L]_+ \\ > & P_0 - K_L - q_H[P_H - K_L]_+ - (1 - q_H)[P_L - K_L]_+ \\ = & P_0 - K_L - q_H[P_H - K_L] - (1 - q_H)[P_L - K_L] \\ \geq & P_0 - K_H - q_H[P_H - K_H]_+ - (1 - q_H)[P_L - K_H]_+ \\ = & P_0 - K_H - F_H. \end{aligned}$$

□

Implausible Equilibrium: Non-monotone beliefs

Suppose that **both** types **trade in period 0**: $S_H = S_L = 1$.

Then period 1 beliefs π_L, π_H are not well-defined.

We **choose off-path beliefs** π_L, π_H to sustain early trade in eq:

Set $\pi_L = \pi_H = 0$. Then $P_L = P_H = V_L$ and $P_0 = \mathbb{E}[V_\theta]$.

Then prices are **consistent** with beliefs and prices satisfy **zero profit**.

Period 0 stopping is **optimal**.

But **unintuitive**: H has lower gains from early trade.

Rule out via **monotone belief refinement**:

Definition (monotone belief refinement)

If $S_L, S_H = 1$, period 1 beliefs are consistent with some $S_L \geq S_H \to 1$.

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Lemma

In any price- and belief-monotone equilibrium, $S_L = 1 \Rightarrow S_H = 0$.

Proof.

Since $S_L \geq S_H$, it holds that

$$\mathbb{E}[\pi_X|\theta = H] = q_H\pi_H + (1 - q_H)\pi_L$$

$$\geq q_H \frac{\pi_0 q_H}{\pi_0 q_H + (1 - \pi_0)q_L} + (1 - q_H)\frac{\pi_0(1 - q_H)}{\pi_0(1 - q_H) + (1 - \pi_0)(1 - q_L)}$$

$$\overset{\star}{>} \pi_0.$$

$\star$ *Write* $q_H = q_L + \triangle q$ *— differentiate in* $\triangle q$ *— require* $q_H > \frac{1}{2}$

Hence, in equilibrium $\mathbb{E}[P_X|\theta = H] > P_0$. Then there exists a profitable deviation: type H selling in period 1 at the future prevailing market prices P_H or P_L yields strictly greater expected utility than selling at P_0. □

Delay is Inevitable

Just shown: $S_L < 1 \Rightarrow S_H = 0$ and $S_L = 1 \Rightarrow S_H = 0$.

- **Insight 1**: H never sells in period 0.

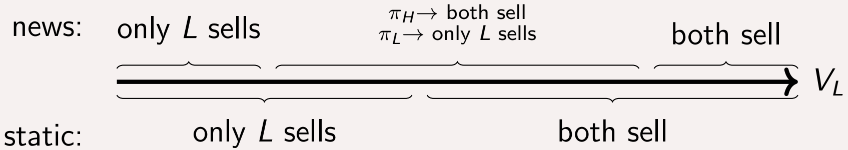
Note: If H trades in equilibrium, then only so in period 1. Hence L also trades in period 1 as prices are higher. $S_L = 0$.

- **Insight 2**: If H trades in period 1, L always sells in period 1, never in period 0.

Welfare Comparison

News changes the equilibrium:

- **Welfare Improvement** News can facilitate trade if previously H does not trade at π_0.
- **Novel market failure** Waiting can result in no trade if news is bad.



Daley and Green (2012): Continuous Time

Set-up: Continuous Time

Time: Continuous, $t \geq 0$, discount rate r.

- Seller: Asset $\theta \in \{L, H\}$, prior $\pi_0 = P(\theta = H)$.
- Payoffs normalized by r: flow and lifetime on same scale.
- Seller payoff (sell at t):

$$(1 - e^{-rt})K_\theta + e^{-rt} rm$$

- Buyer payoff (buy at t): $V_\theta - rm$.

Assumptions:

- $V_H > V_L$, $K_H > K_L$, gains from trade: $V_\theta > K_\theta$.

News Process

News: Brownian diffusion:

$$dX_t = \mu_\theta dt + \sigma dB_t, \quad X_0 = 0$$

- B_t: Standard Brownian motion, $\mathcal{F}_t$-adapted.
- $\mu_H > \mu_L$: Good type drifts higher.

Notation:

- $\phi = \frac{\mu_H - \mu_L}{\sigma}$, $\gamma = \frac{\phi^2}{r}$: News quality.

Beliefs: News Alone

Analyst's Belief (no trading):

$$\pi_t = \frac{\pi_0 f_t^H(X_t)}{\pi_0 f_t^H(X_t) + (1-\pi_0) f_t^L(X_t)}$$

- $f_t^\theta(X_t) = \frac{1}{\sqrt{2\pi t \sigma^2}} \exp\left(-\frac{(X_t - t\mu_\theta)^2}{2t\sigma^2}\right)$.

Likelihood Ratio (no trading):

$$\frac{\pi_t}{1-\pi_t} = \frac{\pi_0 f_t^H(X_t)}{(1-\pi_0) f_t^L(X_t)}$$

$$Z_t \equiv \ln\left(\frac{\pi_t}{1-\pi_t}\right) = \ln\left(\frac{\pi_0}{1-\pi_0}\right) + \ln\left(\frac{f_t^H(X_t)}{f_t^L(X_t)}\right) \equiv \hat{Z}_t$$

- $\hat{Z}_t$: Belief from news alone (no trade info).

Beliefs: News Alone (Derivation)

Compute $\hat{Z}_t$:

$$\ln\left(\frac{f_t^H(X_t)}{f_t^L(X_t)}\right) = \ln\left(\frac{\exp\left(-\frac{(X_t - t\mu_H)^2}{2t\sigma^2}\right)}{\exp\left(-\frac{(X_t - t\mu_L)^2}{2t\sigma^2}\right)}\right)$$

$$= -\frac{(X_t - t\mu_H)^2}{2t\sigma^2} + \frac{(X_t - t\mu_L)^2}{2t\sigma^2}$$

$$= \frac{1}{2t\sigma^2}\left[(X_t - t\mu_L)^2 - (X_t - t\mu_H)^2\right]$$

$$= \frac{1}{2t\sigma^2}\left[2X_t t(\mu_H - \mu_L) + t^2(\mu_H^2 - \mu_L^2)\right] = \frac{\mu_H - \mu_L}{\sigma^2}X_t - \frac{\mu_H^2 - \mu_L^2}{2\sigma^2}t$$

$$\hat{Z}_t = \hat{Z}_0 + \frac{\phi}{\sigma}X_t - \frac{(\mu_H + \mu_L)\phi}{2\sigma}t, \qquad \boxed{d\hat{Z}_t = -\frac{(\mu_H + \mu_L)\phi}{2\sigma}dt + \frac{\phi}{\sigma}dX_t}$$

Beliefs: News Alone

Beliefs in the absence of trade follow an Ito process:

$\hat{Z}_t$ evolves according to

$$d\hat{Z}_t = -\frac{1}{2\sigma}(\mu^H + \mu^L)\phi dt + \frac{1}{\sigma}\phi dX_t = \begin{cases} \frac{1}{2}\phi^2 dt + \phi dB_t & \text{if } \theta = H \\ -\frac{1}{2}\phi^2 dt + \phi dB_t & \text{if } \theta = L. \end{cases}$$

- H expects that $\hat{Z}_t$ will rise,
- L expects that $\hat{Z}_t$ will fall.

Market Beliefs

Market Belief (no trade by t):

$$\pi_t = \frac{\pi_0 f_t^H(X_t)(1 - S_{t^-}^H)}{\pi_0 f_t^H(X_t)(1 - S_{t^-}^H) + (1 - \pi_0) f_t^L(X_t)(1 - S_{t^-}^L)}$$

- $S_{t^-}^\theta = \lim_{s \uparrow t} S_s^\theta$: Pre-trade probability.

Log-Likelihood:

$$Z_t = \ln\left(\frac{\pi_0 f_t^H(X_t)(1 - S_{t^-}^H)}{(1 - \pi_0) f_t^L(X_t)(1 - S_{t^-}^L)}\right)$$

$$Z_t = \underbrace{\ln\left(\frac{\pi_0}{1 - \pi_0}\right) + \ln\left(\frac{f_t^H(X_t)}{f_t^L(X_t)}\right)}_{\hat{Z}_t} + \underbrace{\ln\left(\frac{1 - S_{t^-}^H}{1 - S_{t^-}^L}\right)}_{Q_t}$$

- $\hat{Z}_t$: News-driven belief.
- Q_t: Trading signal.

Seller's Problem: Optimal Stopping

Goal: Maximize:

$$F_\theta = \sup_\tau \mathbb{E}\left[\int_0^\tau e^{-rs} rK_\theta ds + e^{-r\tau} W_\tau\right]$$

- τ: Stopping time, $\mathcal{F}_t$-adapted.
- W_t: Market price at t (scaled by r).

Continuation Value (at t, $Z_t = z$):

$$F_\theta(z) = \mathbb{E}\left[\int_t^\tau e^{-r(s-t)} rK_\theta ds + e^{-r(\tau-t)} W_\tau \middle| Z_t = z\right]$$

- Time-stationary: Depends on z, not t.

Equilibrium Definition

Definition (Equilibrium)

Tuple $(Z_t, S_t^L, S_t^H, F_t^L, F_t^H, W_t)$, $\mathcal{F}_t$-adapted:

- **Optimal Stopping**: S_t^θ solves F_θ.
- **Consistent Beliefs**: $Z_t = \hat{Z}_t + Q_t$.
- **Zero Profit**: If trade at t, $W_t = \mathbb{E}[V_\theta | \text{trade}]$.
- **No (unrealized) Deals**: If $S_{t^-}^L < 1$, $F_L(Z_t) \geq V_L$. If $S_{t^-}^H(Z_t) < 1$, $F_H(Z_t) \geq \frac{Z_{t-}}{1+Z_{t-}} V_H + \frac{1}{1+Z_{t-}} V_L$.

$\alpha - \beta$ Equilibrium

An equilibrium candidate:

Thresholds: $\alpha < \beta$.

- $Z_t \in (\alpha, \beta)$: No trade.
- $Z_t = \beta$: Both sell, $W_t = \psi(\beta) = \frac{\beta}{1+\beta} V_H + \frac{1}{1+\beta} V_L$.
- $Z_t = \alpha$: L randomizes, $W_t = V_L$, α reflects.

Reflection:

$$Q_t = \max \left\{ \alpha - \inf_{s \leq t} \hat{Z}_s, 0 \right\}$$

- L adjusts S_t^L to keep $Z_t \geq \alpha$.

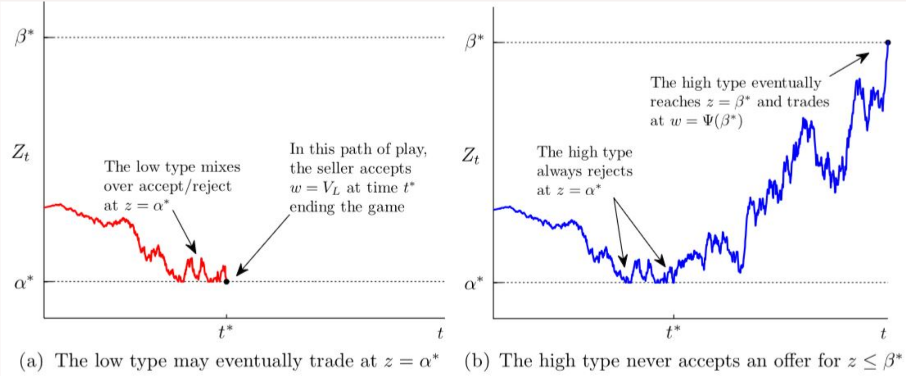


FIGURE 3.—Equilibrium dynamics for a fixed sample path.

Value Functions: Derivation (1)

Dynamic Programming ($z \in (\alpha, \beta)$):

$$F_\theta(Z_t) = \int_t^{t+\Delta t} e^{-r(s-t)} r K_\theta ds + e^{-r\Delta t}\mathbb{E}[F_\theta(Z_{t+\Delta t})|Z_t] + \mathcal{O}(\Delta t^2)$$

- Expand (fundamental theorem of calculus):
  $e^{-r\Delta t}\mathbb{E}[F_\theta(Z_{t+\Delta t})|Z_t] =$
  $e^{-r\Delta t}\left\{F_\theta(Z_t) + \mathbb{E}\left[\int_t^{t+\Delta t} dF_\theta(Z_s)|Z_t\right]\right\}.$

Itô's Lemma: Time-stationary ($\frac{\partial F_\theta}{\partial t} = 0$):

$$dF_\theta(Z_s) = F'_\theta(Z_s)dZ_s + \frac{1}{2}F''_\theta(Z_s)\underbrace{(dZ_s)^2}_{=\phi^2 dt}$$

Recall:

$$\hat{Z}_t = \begin{cases} \frac{1}{2}\phi^2\, dt + \phi\, dB_t & \text{if } \theta = H \\ -\frac{1}{2}\phi^2\, dt + \phi\, dB_t & \text{if } \theta = L \end{cases}$$

Ito's lemma

What to make of a new stochastic process $Y_t = g(t, X_t)$ given an existing Ito process X_t?

Y_t is also an Ito process. its law dY_t:

$$dY_t = \frac{d}{dt}g(t, X_t)dt + \frac{d}{dx}g(t, X_t)dX_t + \frac{1}{2}\frac{d^2}{dx^2}g(t, X_t) \cdot (dX_t)^2.$$

$(dX_t)^2 = dX_t \cdot dX_t$ is computed according to
$dt \cdot dt = dt \cdot dB_t = dB_t \cdot dt = 0$, $dB_t \cdot dB_t = dt$.

Value Functions: Derivation (2)

Dynamic Programming $(Z_t \in (\alpha, \beta))$:

$$F_\theta(Z_t) = \int_t^{t+\triangle t} e^{-r(s-t)} rK_\theta ds + e^{-r\triangle t}\left\{F_\theta(Z_t) + \mathbb{E}\left[\int_t^{t+\triangle t} dF_\theta(Z_s)\Big|Z_t\right]\right\} + \mathcal{O}(\Delta t^2)$$

Substitute and Limit:

$$0 = \lim_{\Delta t \to 0} \frac{1}{\Delta t}\left[\int_t^{t+\Delta t} e^{-r(s-t)} rK_\theta ds + (e^{-r\Delta t} - 1)F_\theta(Z_t) + e^{-r\Delta t}\int_t^{t+\Delta t} \mathbb{E}[dF_\theta(Z_s)|Z_t]\right]$$

Expectations:

$$\mathbb{E}[dF_H(Z_s)|Z_t] = \left(F_H'(Z_s)\frac{1}{2}\phi^2 + \frac{1}{2}F_H''(Z_s)\phi^2\right) dt$$

$$\mathbb{E}[dF_L(Z_s)|Z_t] = \left(-F_L'(Z_s)\frac{1}{2}\phi^2 + \frac{1}{2}F_L''(Z_s)\phi^2\right) dt$$

Limit: $0 = rK_\theta - rF_\theta(Z_t) + \left(\pm F_\theta'(Z_s)\frac{1}{2}\phi^2 + \frac{1}{2}F_\theta''(Z_s)\phi^2\right)$

Boundary Conditions

ODEs:

$$rF_L(z) = rK_L - \frac{\phi^2}{2}(F_L'(z) - F_L''(z))$$

$$rF_H(z) = rK_H + \frac{\phi^2}{2}(F_H'(z) + F_H''(z))$$

$$\text{or } \boxed{F_\theta(z) = K_\theta + \frac{\gamma}{2}(F_\theta'(z) \pm F_\theta''(z))}, \quad \gamma = \frac{\phi^2}{r}$$

Value Matching:

- $F_H(\beta) = F_L(\beta) = \psi(\beta)$,
- $F_L(\alpha) = V_L$.

Smooth Pasting:

- $F_H'(\alpha) = 0$ (reflection — mysterious to me),
- $F_H'(\beta) = \psi'(\beta)$, $F_L'(\alpha) = 0$ (optimal stopping).
- Pins α, β uniquely.

Proof: $F_H'(\beta) = \psi'(\beta)$

Consider the alternatives:

- If $F_H'(\beta) < \psi'(\beta)$, then:
  - $F_H(\beta) - F_H(\beta - \epsilon) < \psi(\beta + \epsilon) - \psi(\beta)$ for small $\epsilon > 0$.
  - Since $F_H(\beta) = \psi(\beta)$, any convex combination of $F_H(\beta - \epsilon)$ and $\psi(\beta + \epsilon)$ is better than $F_H(\beta)$.
  - This implies stopping at β cannot be optimal for the type H seller.
- If instead $F_H'(\beta) > \psi'(\beta)$, then:
  - $F_H(\beta) - F_H(\beta - \epsilon) > \psi(\beta) - \psi(\beta - \epsilon)$ for small $\epsilon > 0$.
  - This violates the no-deals condition: the H type seller would be better off selling at price $\psi(\beta - \epsilon)$ when reaching belief $\beta - \epsilon$, which would be acceptable to buyers.

Proof: $F_L'(\alpha) = 0$

Follow analogous reasoning:

- If $F_L'(\alpha) > 0$, rejecting at α and possibly seeing market beliefs slide below α gives a higher payoff than selling.
- If $F_L'(\alpha) < 0$, the no-deals condition is upset.

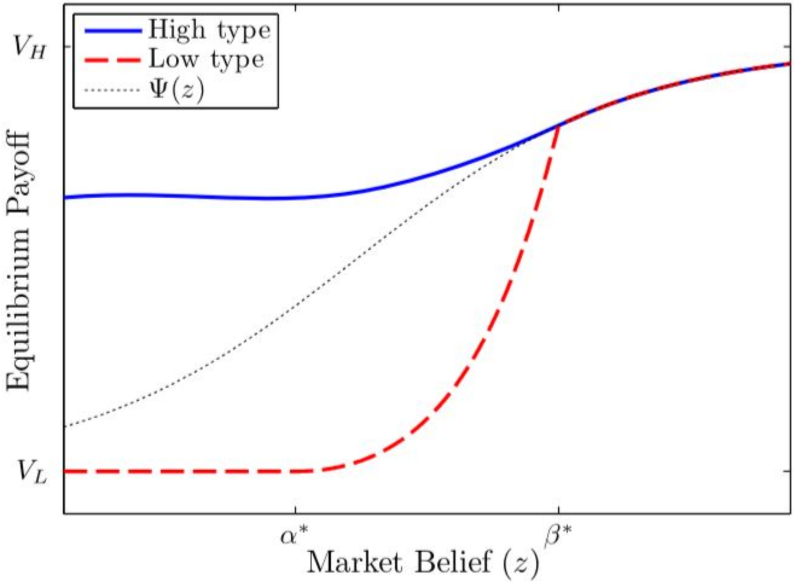


FIGURE 2.—Equilibrium value functions under $\Xi(\alpha^*, \beta^*)$.

Further Results

Existence: $\alpha - \beta$ equilibrium exists if:

- γ large (strong news),
- $V_L > K_H$ (severe lemons problem).

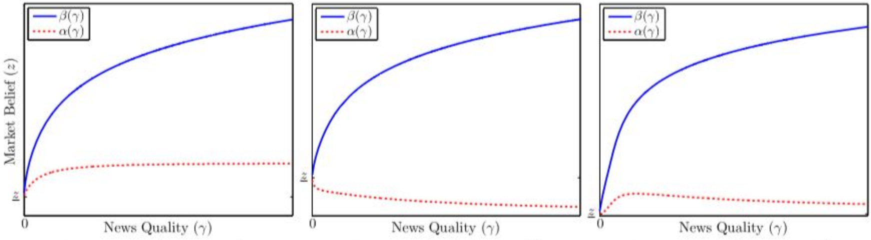
Limit: As $\gamma \to 0$ (no news):

- Trade at $t = 0$, prices match Akerlof.

Uniqueness: If $V_L > K_H$, $\alpha - \beta$ is unique (monotone Q_t).

Multiplicity: If $V_L < K_H$, can construct another $\alpha' - \beta'$ equilibrium where both trade at α'.

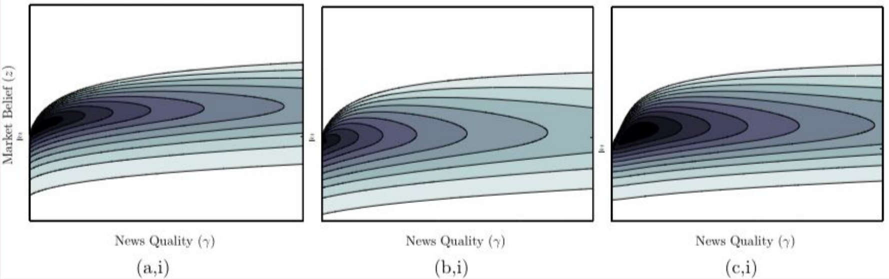
Comparative Statics



(a) $K_L = -\frac{1}{2}$, $K_H = \frac{1}{4}$ (b) $K_L = -\frac{1}{10}$, $K_H = \frac{1}{4}$ (c) $K_L = -\frac{1}{20}$, $K_H = \frac{1}{20}$

FIGURE 4.—Equilibrium boundaries as they depend on γ for three different values of (K_L, K_H), with $V_L = 0, V_H = 1$, and γ ranging from 0 to 20.

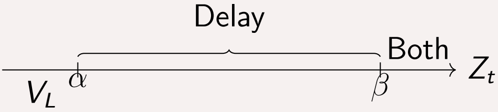
Comparative Statics



Conclusion

Static vs. Dynamic:

- **Improvement**: News enables H trade.
- **New Failure**: Delay for intermediate Z_t.



Language How to Conceptualize Dynamic Trading

Next Steps

- Information Acquisition
- Divisible Goods (nonexclusive competition)