Countervailing Vertical Contracting

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Motivation

- Contracts can serve as **commitment devices**.
- Under **asymmetric information**, the degree of commitment given is unclear.
- **Question:** What mechanisms can ex-ante contracts implement? How much commitment do they provide?

Study question in a bilateral trade model.

- Focus on weak suppliers vs. strong buyers (e.g., drug manufacturers give licenses to distributors who then bargain with large retailers).

Overview

- Private-cost supplier seeks to sell a good to a private-value buyer.
- Buyer posts TIOLI prices to supplier.

Before observing cost, supplier must obtain third-party license.

Main Result — A Three-Way Equivalence

Trade mechanism is IC and IR for buyer and DSIC for supplier	$\Longleftrightarrow$	Implementable through ex-ante contracting	$\Longleftrightarrow$	Implementable by supplier who commits to a price-acceptance strategy

Model

A private-value buyer, $\mathcal{B}$, posts TIOLI prices to a private-cost supplier, $\mathcal{S}$.

- Common prior $c \sim G$, $v \sim F$, with $v \perp c$ and F, G smooth. Full-support densities, g and f over $[\underline{c}, \overline{c}]$ and $[\underline{v}, \overline{v}]$, respectively.

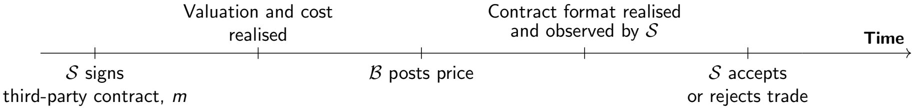
Before observing cost, $\mathcal{S}$ signs an observable and irrevocable contract.

- Contract specifies (possibly randomised) payments from $\mathcal{S}$ to a third party.
- Contract conditions on whether trade occurs $(x = 1)$ or not $(x = 0)$ and price posted by buyer, p. Represented by mapping

$$\{0, 1\} \times \mathbb{R}_+ \ni (x, p) \mapsto m(x, p) \in \Delta(\mathbb{R})$$

- Let $\mathbb{M}$ be the set of all measurable contracts with downstream equilibria.
- Define $\underline{\mathbb{M}}$ as set of realised contracts from $\mathbb{M}$, post randomisation.

Strategies



Buyer observes the contract format, m, but not its realisation before posting price. Supplier observes posted price and contract's realisation before deciding whether to trade.

- $\mathcal{B}$ picks price schedule $p : [\underline{v}, \overline{v}] \times \mathbb{M} \to \Delta(\mathbb{R}_+)$.
- $\mathcal{S}$ chooses price-acceptance rule $a : [\underline{c}, \overline{c}] \times \mathbb{R}_+ \times \underline{\mathbb{M}} \to \Delta(\{0, 1\})$.

Payoffs and Equilibrium

- Given contract m signed, payoffs are,

$$\pi_{\mathcal{B}}(x, p\ ; v) = x(v - p)$$
$$\pi_{\mathcal{S}}(x, p, m\ ; c) = x(p - c) - m(x, p)$$

- Define supplier trade surplus as $\hat{\pi}_{\mathcal{S}}(x, p\ ; c) := x(p - c)$.

An m-equilibrium is,

- A price-acceptance strategy $a : [\underline{c}, \overline{c}] \times \mathbb{R}_+ \times \underline{\mathbb{M}} \to \Delta(\{0, 1\})$ for $\mathcal{S}$.
- A price schedule $p : [\underline{v}, \overline{v}] \times \mathbb{M} \to \Delta(\mathbb{R}_+)$ for $\mathcal{B}$.

Such that a and p are sequentially rational given m and contract is *acceptable:* $\mathbb{E}_{(p,a)}[\pi_{\mathcal{S}}] \geq 0$.

Simplifying the Contract Space

Lemma 1

Without loss of generality, we can restrict attention to **two-part contracts**,

$$m(x,p) = xk(p) + T$$

which are always accepted by $\mathcal{S}$ where

- T is a **Fixed Fee** paid irrespective of downstream trade outcomes.
- $k : \mathbb{R}_+ \to \Delta(\mathbb{R})$ is a (randomised) **Royalty Payment** paid if trade occurs and can depend on p.

Proof Sketch: Focus on two-part restriction. Decompose any m as

$$m(x,p) = x(m(1,p) - m(0,p)) + m(0,p) =: xk(p) + T(p)$$

- Price-acceptance decision of $\mathcal{S} \perp T(p)$, can flatten to $T = \mathbb{E}[T(p)]$.

Trade Mechanisms

Following Myerson-Satterthwaite (1983), an outcome of bilateral trade is described by a direct mechanism

$$(q,t):[\underline{c},\overline{c}]\times[\underline{v},\overline{v}]\to[0,1]\times\mathbb{R}$$

We ask when one can achieve a mechanism through contracting,

Contract Implementation

An outcome (q,t) is contract implementable if there exists $m\in\mathbb{M}$ and m-equilibrium (p,a) such that

$$q(c,v)=\mathbb{E}_{(p,a)}[a(c,p(v))]$$
$$t(c,v)=\mathbb{E}_{(p,a)}[p(v)q(c,v)]$$

and is further contract implementable without outside subsidy if $\mathbb{E}_{(p,a)}[m(a(p),p)]\geq 0$.

Contract-Implementable Outcomes

Lemma 2

Outcome (q,t) is contract implementable if and only if

1. $q(c,v)$ is non-increasing in c for each $v \in [\underline{v},\overline{v}]$.

2a. $\int_{\underline{c}}^{\overline{c}} q(c,v)dG$ is non-decreasing in $v \in [\underline{v},\overline{v}]$.

2b. $\int_{\underline{c}}^{\overline{c}} t(c,v)dG = \int_{\underline{c}}^{\overline{c}} \left[vq(c,v) - \underline{v}q(c,\underline{v}) + t(c,\underline{v}) - \int_{\underline{v}}^{v} q(c,x)dx\right] dG$

3. $\int_{\underline{c}}^{\overline{c}} q(c,\underline{v})v - t(c,\underline{v})dG \geq 0$.

An outcome (q,t) is *contract-implementable without outside subsidy* if and only if conditions 1-3 hold and it is *profitable*:

4. $\int_{\underline{c}}^{\overline{c}} \int_{\underline{v}}^{\overline{v}} t(c,v) - q(c,v)c \; dFdG \geq 0$.

Condition 1 is DSIC for supplier. 2a, 2b, and 3 are interim IC and IR for buyer. 4 is ex-ante participation for supplier.

Contract-Implementable Outcomes

Proof: **Contract Implementable $\Longrightarrow$ Conditions**

- (q, t) contract implementable by contract m and (p, a) an m-equilibrium.
- Buyer IC+IR hold as equilibrium.
- $\mathcal{S}$ makes price-acceptance decision **after** observing price, $\mathcal{S}$'s DSIC holds.
- If contract implementable without outside subsidy, $\mathbb{E}_{(p,a)}[m(a(p), p)] \geq 0$.
- Can increase $\mathbb{E}_{(p,a)}[m(a(p), p)]$ by raising fixed fee T to make supplier acceptability condition bind.
- If binds, $\mathbb{E}_{(p,a)}[m(a(p), p)]$ equal to supplier's ex-ante expected surplus.
- So,

$$\int_{\underline{c}}^{\overline{c}} \int_{\underline{v}}^{\overline{v}} t(c, v) - q(c, v)c \; dFdG \geq \mathbb{E}_{(p,a)}[m(a(p), p)] \geq 0$$

Contract-Implementable Outcomes

Proof: **Conditions $\implies$ Contract Implementable**

- Let (q, t) be any outcome satisfying supplier DSIC and buyer interim IR+IC. Constructively define contract $m(x, p) = T + xk(p)$ and m-equilibrium (p, a) implementing (q, t).
- By buyer IC, the set of types who never trade is interval $[\underline{v}, \tilde{v})$. Set $k(p)$ very large ($> \overline{v}$) for $p < \tilde{v}$ to prevent trade.
- For any $v > \tilde{v}$, define $p(v) = \frac{\int_{\underline{c}}^{\overline{c}} t(c,v)}{\int_{\underline{c}}^{\overline{c}} q(c,v)}$.
- c-type accepts trade at $p(v)$ with probability $\mathbb{P}(k(p(v)) \leq p(v) - c)$.
- Set $\mathbb{P}(k(p(v)) \leq p(v) - c) = q(c, v)$ and vary over $c \in [\underline{c}, \overline{c}]$. Defines CDF of $k(p(v))$, well-defined as $q(c, v)$ non-increasing in c for given v.
- Such k implements (q, t). If supplier ex-ante IR holds, set T to bind supplier contract acceptance $\rightarrow$ no outside subsidy needed.

Supplier Commitment

- Envisage commitment version of model. Replace contract with commitment strategy $\alpha : [\underline{c}, \overline{c}] \times \mathbb{R}_+ \to [0,1]$.
- $\alpha(c,p)$ is probability supplier with cost c accepts price offer p.

Monotone Commitment

Say commitment strategy α is **monotone** if for $c > c'$, $\alpha(c,p) \leq \alpha(c',p)$.

Monotone-Commitment Implementation

Outcome (q,t) is *monotone-commitment-implementable* (and profitable) if there exists pair (α, p_α) with α monotone and p_α a buyer best response to α such that

$$q(c,v) = \mathbb{E}_{(p,\alpha)}[\alpha(c, p_\alpha(v))]; \qquad \text{(Allocation rule)}$$

$$t(c,v) = \mathbb{E}_{(p,\alpha)}[p_\alpha(v) q(c,v)]. \qquad \text{(Transfer rule)}$$

(and $\mathbb{E}_{(p,\alpha)}[\alpha(p)(p-c)]] \geq 0$).

A Three-Way Equivalence

Theorem 1

For any outcome, (q, t), the following statements are equivalent:

(i) (q, t) is contract implementable (without outside subsidy)

(ii) (q, t) is implementable by a mechanism designer who must satisfy ex-interim incentive compatibility and individual rationality for the buyer and ex-post incentive compatibility (and ex-ante individual rationality) for the seller.

(iii) (q, t) is monotone-commitment implementable (and profitable).

- Ignore profitability/without outside subsidy aspect.
- Already shown $(i) \iff (ii)$.
  $(i) \implies (iii)$ follows by setting $\alpha(c, p) = \mathbb{P}(k(p) \leq p - c)$.
- WTS $(iii) \implies (i)/(ii)$. If (q, t) monotone-commitment implementable, supplier DSIC follows by monotonicity. Buyer IC+IR by equilibrium.
- So (q, t) is contract implementable by earlier Lemma.

Myerson-Satterthwaite Mechanisms

MS-Implementation

Outcome (q, t) is **MS-implementable** if it is interim IR + IC for $\mathcal{B}$ and $\mathcal{S}$.

Recall: (q, t) contract implementable without outside subsidy iff interim IR + IC for $\mathcal{B}$, and DSIC and ex-ante IR for $\mathcal{S}$.

Lemma 2

For any MS-implementable outcome, (q, t), there exists a contract implementable outcome which is interim payoff equivalent to (q, t).

Proof: Yang and Yang (2025) show any MS-implementable outcome is interim-payoff equivalent to a mix of 'markup-pooling' outcomes.
Mixes of markup-pooling outcomes are interim IR + IC for $\mathcal{B}$ and DSIC for $\mathcal{S}$.
Aligns with BIC≅DSIC results (Manelli & Vincent, 2010, inter alia.)

Efficiency

Theorem 2

Ex-post efficient trade is contract implementable without outside subsidy where:

1. The contract $\langle k^*_{\mathcal{B}}, T^*_{\mathcal{B}} \rangle$ with

$$k^*_{\mathcal{B}}(p) := \begin{cases} p - (p_{k^*_{\mathcal{B}}})^{-1}(p) & p \in [\mathbb{E}_c[c \mid c \leq \underline{v}], \mathbb{E}[c]] \\ \overline{v} & \textit{otherwise} \end{cases}$$

$$T^*_{\mathcal{B}} := \mathbb{E}_{c,v}[\mathbb{1}_{\{v \geq c\}}(v - c)]$$

is signed by $\mathcal{S}$;

2. The pricing decision of $\mathcal{B}$ given k^* is

$$p_{k^*_{\mathcal{B}}}(v) := \mathbb{E}_c[\max\{c, \underline{v}\} \mid c \leq v];$$

3. The acceptance decision of $\mathcal{S}$ following any k and p is given by

$$a_k(c, p) := \mathbb{1}_{\{c \leq p - k(p)\}}.$$

Efficiency

Because $p_{k^*_{\mathcal{B}}}(v) - k(p_{k^*_{\mathcal{B}}}(v)) = v$. expected surplus of c-supplier type is

$$\int_{\underline{v}}^{\overline{v}} (p_{k^*_{\mathcal{B}}}(v) - k(p_{k^*_{\mathcal{B}}}(v)) - c)\mathbb{1}_{\{c \leq v\}} dF = \int_{\underline{v}}^{\overline{v}} (v - c)\mathbb{1}_{\{c \leq v\}} dF$$

- Each cost type is *locally* residual claimant on trade surplus.
- Efficiency compatible with balanced budget as relaxed supplier's interim IR.
- Decentralises an AGV mechanism through contracting.

THANK YOU

Supplier-Optimal Contract

To what extent may ex-ante contract/commitment countervail the buyer's bargaining power?

- Suppose F has increasing hazard rate and G has decreasing reverse hazard.
- Define $\psi_{\mathcal{B}}(v) = v - \frac{1-F(v)}{f(v)}$.
- If supplier has price-posting power, sets

$$p_{\mathcal{S}}(c) = \psi_{\mathcal{B}}^{-1}(c) = \inf\{p \in \mathbb{R} \mid \psi_{\mathcal{B}}(p) \geq c\}$$

Compare supplier's monopoly outcome with that under royalties.

- Let $\hat{\pi}_{\mathcal{S}}(\text{monopoly})$ be supplier payoff when it sets prices.
- Let $\hat{\pi}_{\mathcal{S}}(\text{royalty})$ be supplier payoff under its optimal contract.

Supplier-Optimal Contract

Theorem 2

The supplier-optimal contract is payoff-equivalent to the supplier posting prices $\hat{\pi}_{\mathcal{S}}(\text{monopoly}) = \hat{\pi}_{\mathcal{S}}(\text{royalty})$ with:

1. The contract $\langle k^*, T^* \rangle$ with

$$k^*(p) := \begin{cases} p - \psi_{\mathcal{B}}((p_{k^*})^{-1}(p)) & p \in [p^{\mathcal{S}}(\underline{c}), \mathbb{E}[p^{\mathcal{S}}(c)]] \\ \overline{v} & \textit{otherwise} \end{cases}, \quad T^* := 0$$

   is signed by $\mathcal{S}$;

2. The pricing decision of $\mathcal{B}$ given k^* is

$$p_{k^*}(v) := \mathbb{E}[p^{\mathcal{S}}(c) \mid p^{\mathcal{S}}(c) \leq v]$$

3. The acceptance decision of $\mathcal{S}$ following any k and p is given by

$$a_k(c, p) := \mathbb{1}\{c \leq p - k(p)\}.$$

Supplier-Optimal Contract

Ex-ante contracting completely overturns buyer's bargaining power.

- Buyer's price schedule increasing in value.
- Optimal royalty scheme is decreasing in price posted.
- Expected royalty payments are zero.

Contract encourages rejection of low-price offers and subsidises acceptance of high-price offers.

- Pushes buyer prices upwards. Screen buyer's by adjusting probability of trade.
- Zero expected royalties $\implies$ cross-subsidisation of cost types.

Ex-Ante Pareto Frontier

Consider contracts without outside subsidy which are acceptable to the supplier. Seek to characterise the ex-ante Pareto frontier.

- Weight γ on buyer surplus, $1 - \gamma$ on supplier surplus.
- Define $\psi_{\mathcal{B}}(v \mid \gamma) = v + \min\left\{0, \frac{2\gamma-1}{1-\gamma}\right\} \cdot \frac{1-F(v)}{f(v)}$
- Say an outcome is γ-*maximal* if it maximises the γ convex combination of surpluses.

Assume F has increasing hazard rate — $\frac{1-F(v)}{f(v)}$ increasing — so $\psi_{\mathcal{B}}(\cdot \mid \gamma)$ is strictly increasing for all γ.
Also assume G has decreasing reverse hazard rate: g/G decreasing

Ex-Ante Pareto Frontier

Theorem 3

For each $\gamma \in [0, 1]$, the γ-maximal outcome is implementable where:

- The royalty contract k^*_γ defined by

$$k^*_\gamma(p) := \begin{cases} p - \psi_{\mathcal{B}}((p_{k^*_\gamma})^{-1}(p) \mid \gamma) & p \in [\psi_{\mathcal{B}}^{-1}(\underline{c} \mid \gamma), \mathbb{E}[\psi_{\mathcal{B}}^{-1}(c \mid \gamma)]] \\ \overline{v} & \textit{otherwise} \end{cases}$$

  is signed by $\mathcal{S}$;
- The pricing decision of $\mathcal{B}$ given k^* is

$$p_{k^*_\gamma}(v) := \mathbb{E}_c[\max\{\psi_{\mathcal{B}}^{-1}(c \mid \gamma), \underline{v}\} \mid c \leq \psi_{\mathcal{B}}(v \mid \gamma)];$$

- The acceptance decision of $\mathcal{S}$ following any k and p is given by

$$a_{k^*_\gamma}(c, p) := \mathbb{1}_{\{c \leq p - k(p)\}}.$$

Ex-Ante Pareto Frontier

Remarks on the frontier:
In any equilibrium, the expected royalty payment is always negative and equal to

$$\left[\min\left\{0, \frac{1-2\gamma}{1-\gamma}\right\} - 1\right] \cdot \int_{\psi_{\mathcal{B}}^{-1}(\underline{c}|\gamma)}^{\overline{v}} G(\psi_{\mathcal{B}}(x \mid \gamma))[1-F(x)]dx \leq 0$$

- Negative royalties are general feature.
- Negative royalties subsidise supplier for accepting prices.
- Implies fixed fee is really a fee to supplier: $T \geq 0$.

The price schedule and royalty scheme have an intuitive shape.

- For any point on the frontier, the price schedule of $\mathcal{B}$ is increasing in value.
- For any point on the frontier, the royalty scheme is decreasing in price.
- Higher prices associated with higher expected surplus for buyer and supplier surplus. Try to encourage trade at these prices.

Sub-optimality of Posted Prices

Buyer posts prices to supplier — meant to model bargaining power.
Should formally allow buyer to choose the trading mechanism.

- Suppose buyer picks mechanism

$$q_{\mathcal{B}}, t_{\mathcal{B}} : [\underline{c}, \overline{c}] \times [\underline{v}, \overline{v}] \times [0,1] \to \{0,1\} \times \mathbb{R}$$

  where $q_{\mathcal{B}}(c, v; \omega)$ and $t_{\mathcal{B}}(c, v; \omega)$ are *ex-post* with ω representing realisation of randomisation.

- Define a contract as a function $m : \{0,1\} \times \mathbb{R} \to \mathbb{R}$ where $m(q_{\mathcal{B}}(c, v; \omega), t_{\mathcal{B}}(c, v; \omega))$ represents how much $\mathcal{S}$ must pay given ex-post realisation.
- Payoffs are

$$\pi_{\mathcal{B}}(q_{\mathcal{B}}, t_{\mathcal{B}}; c, v, \omega) = q_{\mathcal{B}}(c, v; \omega)v - t_{\mathcal{B}}(c, v; \omega)$$
$$\pi_{\mathcal{S}}(q_{\mathcal{B}}, t_{\mathcal{B}}; c, v, \omega) = t_{\mathcal{B}}(c, v; \omega) - q_{\mathcal{B}}(c, v; \omega)c - m(q_{\mathcal{B}}(c, v; \omega), t_{\mathcal{B}}(c, v; \omega))$$

Sub-optimality of Posted Prices

Buyer has known value $v = 1$. Supplier cost uniform, $c \sim U[0,1]$.
Supplier has signed a concave royalty contract

$$m(q_{\mathcal{B}}(\cdot), t_{\mathcal{B}}(\cdot)) = \begin{cases} \sqrt{1 - t_{\mathcal{B}}(c, v; \omega)^2} & \text{if } q_{\mathcal{B}}(c, v; \omega) = 1, t_{\mathcal{B}}(c, v; \omega) \in [0, 1] \\ 1 & \text{if } q_{\mathcal{B}}(c, v; \omega) = 1, t_{\mathcal{B}}(c, v; \omega) \notin [0, 1] \\ 0 & \text{o/w} \end{cases}$$

- Buyer may convexify the contract through randomisation in transfers.
- Lowers transfer required to make supplier willing to trade.
- Best posted price mechanism gives buyer 0.15 — best randomised transfer mechanism gives 0.24.
- Posted prices may be suboptimal if contract concave. Have an example of this occuring on Pareto frontier.